RESEARCH DEPARTMENT

Effects of polarization on a medium-frequency sky-wave service, including the case of multi-hop paths

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EFFECTS OF POLARIZATION ON A MEDIUM-FREQUENCY SKY-WAVE SERVICE, INCLUDING THE CASE OF MULTI-HOP PATHS

Research Report No. G-083/2

(1964/3)

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SUMMARY

Ionospheric propagation at medium frequencies depends almost entirely on the ordinary wave which is, in general, elliptically polarized. This report describes coupling losses which occur when this wave is transmitted and received with vertical aerials. It also discusses additional losses caused by changes of polarization when intermediate Earth reflexions occur. Formulae are given permitting the various losses to be estimated; these are necessarily approximate, because they are based on the simple ray-theory approach. By way of example, curves are given showing the calculated losses for one typical frequency and angle of incidence.

1. INTRODUCTION

This report discusses one aspect of propagation loss at medium frequency (m.f.) over long distances, namely the contribution due to what may be termed 'polarization coupling' losses. As explained more fully in the body of the report, these must be considered quite separately from absorption losses. They depend on the form of polarization of the wave, which in turn varies with certain features of the path geometry not normally of any consequence in high frequency (h.f.) propagation.

In a previous report[1] attention was drawn to the polarization coupling loss in the case of a night-time sky-wave service at m.f., assuming that the ordinary wave is the only useful mode of propagation, and that effectively vertically polarized aerials must be used at the transmitting and receiving ends. As an example, the losses were given for a path length of 940 km at the magnetic equator (zero angle of magnetic dip), and shown to be serious for directions of propagation around magnetic east or west. This example, and the formulae given in the report for the more general case, have the limitation that they apply primarily to a single-hop path, i.e. a path which involves just one reflexion from the ionosphere.

The main purpose of the present report is to consider the problem more generally, by including the additional losses that arise in multi-hop paths. This extra attenuation associated with each reflexion at the Earth's surface is caused by the limited degree of coupling between the wave reflected by the ground (or sea) and the ordinary-wave mode on re-entering the ionosphere. It will be shown that this loss can be serious for certain paths not necessarily confined to regions near the magnetic equator.

Besides considering the loss arising from an intermediate Earth reflexion, the present report also deals with the 'terminal loss'. This phenomenon is essentially that considered in the previous report, but it is convenient to re-state the formulae in a form suitable for either single- or multi-hop paths. A limited series of curves is given to illustrate how the various losses may be estimated.

It should be emphasized that the quantitative discussion in this report is based entirely on a simple ray-path model, and assumes that the incident wave is resolved in the ionosphere into ordinary and extraordinary waves, of which the latter is heavily absorbed. The splitting into two modes arises from the presence of the Earth's magnetic field but the treatment in terms of ray-paths, while entirely adequate for most problems at high frequency, is of limited accuracy at medium frequency and the relatively high attenuation of the extraordinary wave may be justified only for frequencies within about 200 kc/s of the gyro-magnetic frequency. Moreover, the formula used for polarization of the wave is that applicable to conditions under which the collision frequency of electrons remains low at all levels of appreciable ionization.* Experimental confirmation of the validity of these concepts for night-time m.f. propagation has been made[2] only for moderately oblique angles of incidence. There is some doubt on theoretical grounds concerning the validity at very oblique angles, and the effective polarization of the downcoming wave may differ considerably from that assumed in this report; hence the propagation losses predicted may be in error particularly at extreme ranges. Although it should now be possible to obtain more accurate results by applying the results of recent experimental and theoretical work on the reflecting properties of the ionosphere at m.f., it is nevertheless considered of value to place on record the effects expected on the basis of the simple theory.

2. PHYSICAL NATURE OF LOSSES

2.1. Concept of Coupling Loss

To distinguish them from absorption losses which are due to progressive attenuation along the propagation path, the type of losses with which we are concerned will be called coupling losses. They arise because any wave incident on the ionosphere will excite the ordinary-wave mode to a degree which depends on how nearly the incident polarization resembles the polarization characteristic of that mode. When the two polarizations are complimentary, e.g. linear polarizations at right-angles or circular polarizations with opposite senses of rotation, there will be no coupling with the ordinary-wave mode.

Under the conditions that apply at night-time, the ordinary-wave suffers little absorption loss while the extraordinary wave, for which the characteristic polarization is complimentary, is generally heavily attenuated in the higher part of the E-region for frequencies in the m.f. broadcasting band. Thus, if there is poor coupling with the ordinary-wave mode, the overall path loss will be great; the actual mechanism by which the incident energy is lost in this case is the efficient excitation of the extraordinary-wave mode at the point of entry into E-region and the subsequent dissipation of the energy in this mode at a higher level. We may contrast this behaviour with that at h.f. where the two modes are generally propagated with very similar path absorptions, which are relatively small provided that the frequency is suitably chosen. Should conditions be such that the polarization incident on the ionosphere couples badly with one mode it will couple well with the other mode; a signal at h.f. will thus be propagated efficiently by one or other mode or, more usually, by both modes carrying waves of comparable strength.

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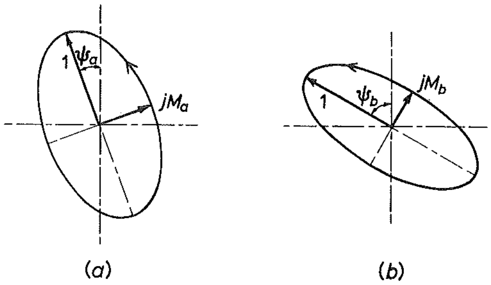
*It may be shown theoretically that collisions between free electrons and gas molecules tend to rotate the plane of polarization of an elliptically polarized wave, at the same time changing its axial ratio. Calculations based on a reasonable upper limit for the collisional frequency[2] show that these effects are unimportant at medium frequencies at all magnetic latitudes during the night.

To simplify the discussion, we will here confine our attention to a single mode of propagation which will be assumed to be the ordinary wave for each hop in the path. If a second mode should be present whose amplitude cannot be neglected, a separate consideration of other contributions to the received signal would become necessary.

2.2. General Case of Coupling between Two States of Polarization

We consider the case where an elliptically polarized wave represented by Fig. 1(a) must be resolved into the component which will excite a mode of propagation with the elliptical polarization of Fig. 1(b). An elliptically polarized wave may be regarded as the resultant of two orthogonal linearly polarized waves in phase quadrature, the planes of polarization of the linear components coinciding with the axes of the ellipse. In Fig. 1 the amplitudes are given according to the usual notation, with real and imaginary values for the in-phase and quadrature components. Looking in the direction of propagation, the ratio of the minor to major axes, M (where $|M| \leqslant 1$), is taken as positive if the sense of rotation is anti-clockwise and negative for the opposite sense. It follows from the Appendix, Section 5.1., that the coupling factor expressed as a power ratio of the induced wave to the incident wave is



(a) (b)

Fig. 1-Notation for polarization ellipse

(a) Incident polarization
(b) Polarization of mode of propagation

$$F = \frac{(1 + M_a M_b)^2 \cos^2(\psi_a - \psi_b) + (M_a + M_b)^2 \sin^2(\psi_a - \psi_b)}{(1 + M_a^2)(1 + M_b^2)} \qquad (1)$$

In the case where the excited mode is linearly polarized in the vertical plane ($M_b = 0$, $\psi_b = 0$) the power coupling factor reduces to

$$F = \frac{\cos^2\psi_a + M_a^2 \sin^2\psi_a}{1 + M_a^2} \qquad (2)$$

This is applicable when an ionospheric wave is incident on a vertical aerial, since the latter responds only to the vertically polarized components. The expression is similar in form when the incident wave is linearly polarized in the vertical plane ($M_a = 0$, $\psi_a = 0$); we simply have equation (2) with M_b, ψ_b replacing M_a, ψ_a. This would apply when a transmission from a vertically polarized aerial is resolved into the ordinary-wave component in the ionosphere.

2.3. Loss Associated with Terminals of Path

It is simplest, first, to consider the loss at the receiving end. Given the parameters of the polarization of the downcoming wave, the power coupling factor F_r is given by expression (2) in which M_a, ψ_a represent the characteristics of the polarization ellipse calculated for the downcoming wave. For the transmitting end we must take a different factor because the characteristic polarization, being a function of the angle between the ray path and the Earth's field, will not be the same. It is possible to calculate the required factor F_t from the polarization parameters, supposing for convenience that the wave is arriving at the transmitting aerial from the ionospheric reflexion point, i.e. travelling along the reciprocal path. This procedure is valid because, for any given part of the path, the form of polarization corresponding to the ordinary wave is the same if the direction of propagation is reversed, subject to a change of sign of both parameters M and ψ to conform to the convention of viewing the polarization in the direction of propagation. Such a change of signs does not affect the form of expression (2).

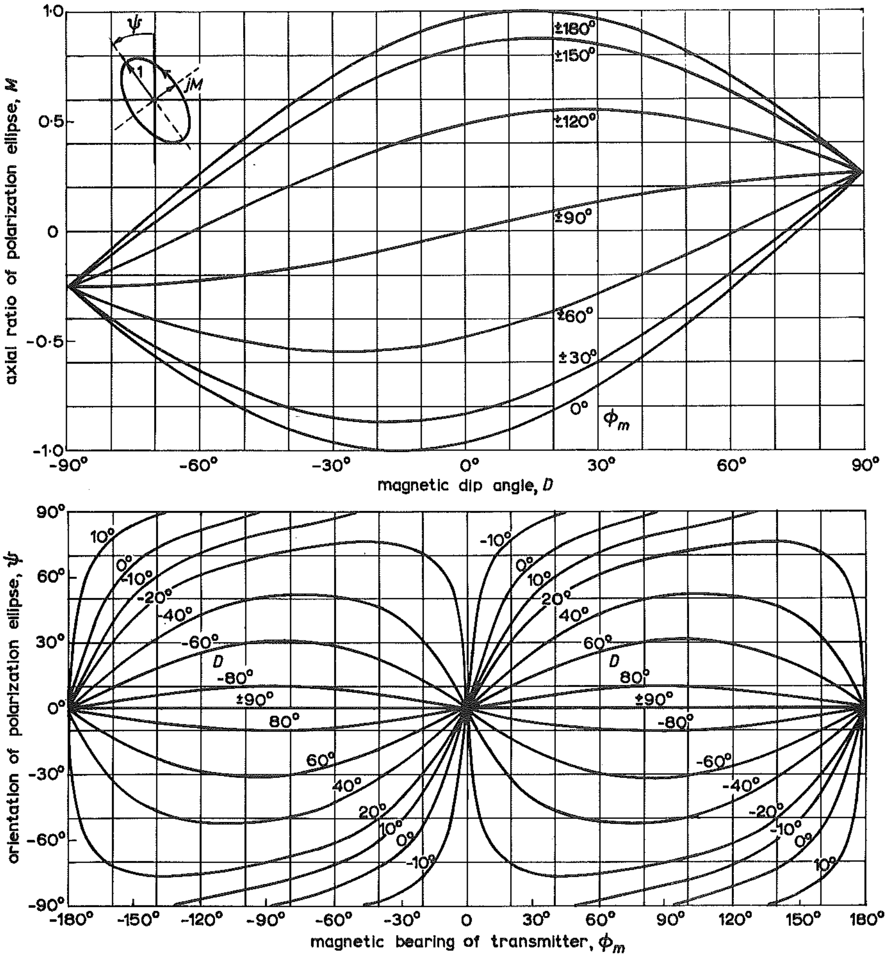


Fig. 2 - Polarization characteristics of ionospheric wave

Angle of incidence at E layer: 75° from vertical
Frequency equal to gyro-magnetic frequency

ϕ_m = magnetic bearing of transmitter
D = magnetic dip angle

As an example, Fig. 2 gives the parameters M and ψ for the special case of the signal frequency equal to the gyro-magnetic frequency,* and an angle of incidence of 75° from the vertical; the collision frequency of electrons is assumed low enough not to affect the polarization. Fig. 3 gives the corresponding factor F based on expression (2). The values are given as a function of the dip angle D at the relevant reflexion point in the ionosphere, and the magnetic bearing ϕ_m of the other terminal, i.e. the bearing relative to magnetic North at the reflexion point. The general formulae are given in the Appendix, Section 5.2.; these are based on those given for night-time conditions by Eckersley and Millington.[2]

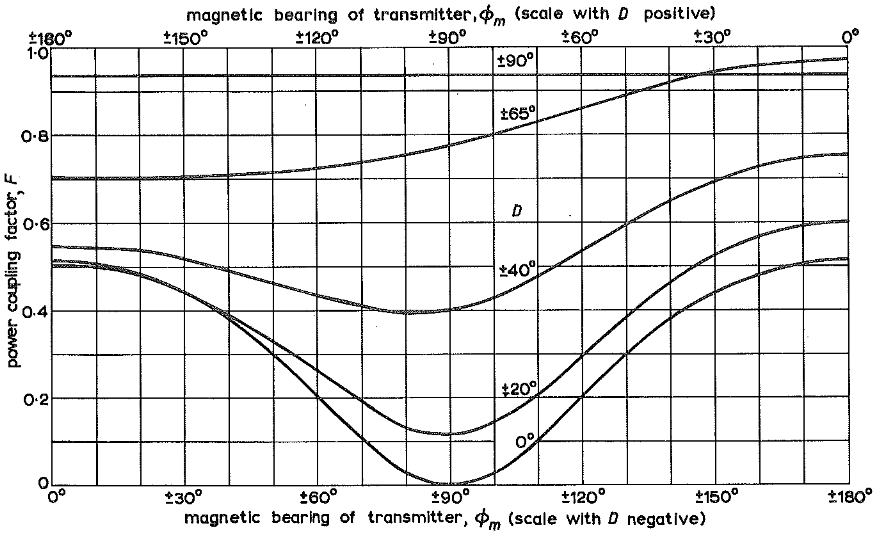


Fig. 3 - Power coupling factor for vertical transmitting or receiving aerials

Angle of incidence at E layer: 75° from vertical
Frequency equal to gyro-magnetic frequency

D = magnetic dip angle. The appropriate horizontal scale should be used depending on the sign of D at the reflexion point. D is positive in the northern hemisphere.

In the case of a single-hop path the product F_tF_r gives the complete coupling loss; moreover, since there is only one reflexion point, single values of dip angle and magnetic bearing apply, so that for a single-hop path the complete loss may be presented in simple graphical form as shown in Fig. 4. This, again, is for the case when the frequency equals the gyro-magnetic frequency, and the angle of incidence in the E-region is 75°.

2.4. Loss Associated with Earth Reflexion

2.4.1. Wave Reflected over Sea

Fig. 5(a) shows the geometry of a two-hop path, with a transmitter at T and a receiving point at R. When the intermediate reflexion point E is over the sea, the

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* The curves contained in this report are approximately correct for frequencies close to the gyro-magnetic frequency. Since the latter always falls within the m.f. band they give a general indication of ionospheric propagation at medium frequencies. For more precise calculations for particular frequencies, the formulae given in Section 5.2. should be used.

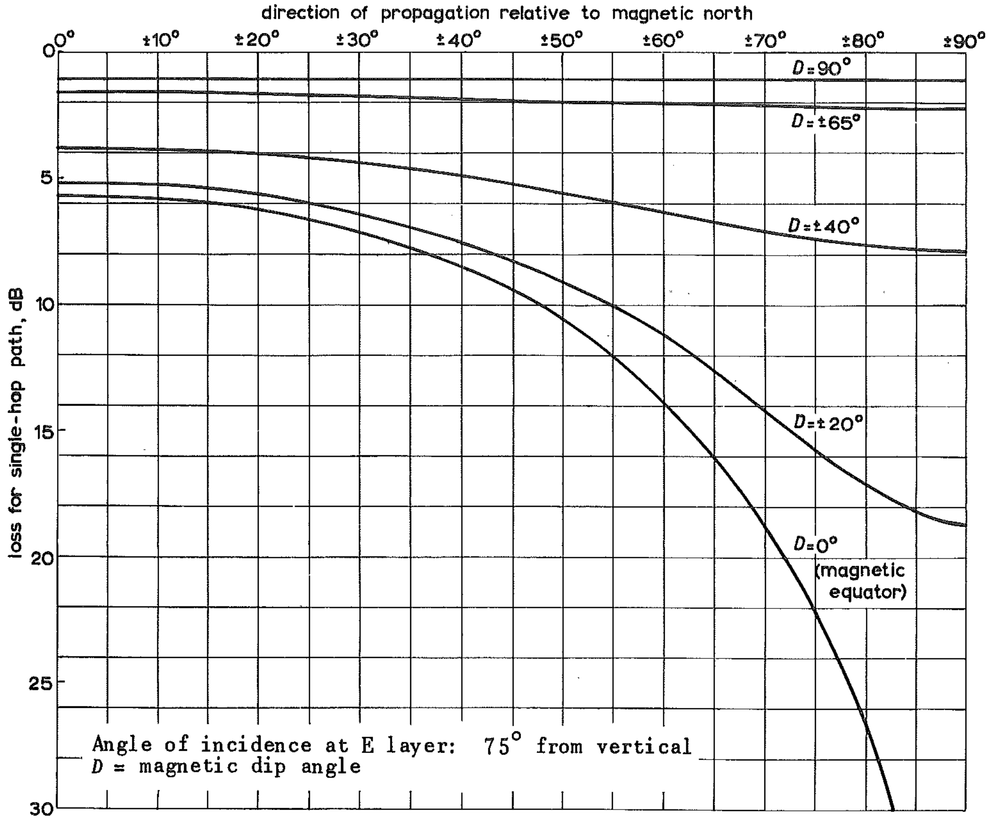

Fig. 4 - Ionospheric propagation loss for single-hop paths with vertical transmitting and receiving aerials

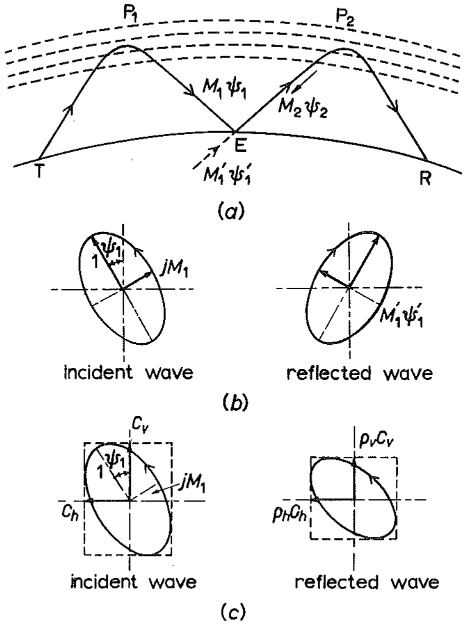

Fig. 5 - The effect of ground reflexion

(a) Geometry of an intermediate reflexion at the Earth's surface

(b) Change of polarization for sea reflexion

(c) Change of polarization for ground reflexion

Earth's surface may be regarded for practical purposes as a perfect conductor. Consequently the phase of the horizontally polarized component of the incident wave is reversed, while the component in the vertical plane is unchanged. The polarization of the reflected wave may thus be taken as the mirror image of that of the incident wave, as shown in Fig. 5(b). It will be seen that the direction of rotation of the polarization has been reversed. Under these conditions we have, referring to Fig. 5(a), $M_1' = -M_1$ and $\psi_1' = -\psi_1$ where $M_1\psi_1$ refer to the polarization of the ordinary wave approaching E from P_1 and M_1', ψ_1' refer to that of the wave reflected from the sea.

Now assume for the moment that a wave is transmitted in the reverse direction from R, and that $M_2\psi_2$ refer to the polarization of an ordinary wave approaching E after reflexion at P_2. The polarization of the ordinary mode for a wave travelling in the forward direction EP_2 as it enters the ionosphere is given by $-M_2$, $-\psi_2$.

The power coupling factor between a wave emerging from the ionosphere at P_1 and re-entering at P_2 after a reflexion from the sea will thus be of the form given by equation (1) with $-M_1$, $-\psi_1$, $-M_2$, $-\psi_2$, substituted for M_a, ψ_a, M_b, ψ_b, namely

$$F_s = \frac{(1 + M_1M_2)^2 \cos^2(\psi_1 - \psi_2) + (M_1 + M_2)^2 \sin^2(\psi_1 - \psi_2)}{(1 + M_1^2)(1 + M_2^2)} \qquad (3)$$

2.4.2. Wave Reflected over Ground

When the Earth reflexion point is over ground it is convenient to regard the downcoming elliptically polarized wave shown in Fig. 5(c) as consisting of a component in the vertical plane given by

$$C_v = \cos\psi_1 + jM_1 \sin\psi_1 \qquad (4)$$

and a horizontal component* given by

$$C_h = \sin\psi_1 - jM_1 \cos\psi_1 \qquad (5)$$

Reflexion by the ground modifies the two linear components by factors ρ_v and ρ_h respectively, which are the appropriate ground reflexion coefficients. Each of the modified components will couple with the ordinary mode in the upgoing wave, giving rise to two contributions to the resultant amplitude. The detailed working is given in the Appendix, Section 5.1. To obtain the power coupling factor, F_g, in terms of the polarization characteristics M_1, ψ_1 of the incident downcoming wave, and M_2, ψ_2 of the wave traversing the reciprocal path down towards E as shown in Fig. 5(a), we take the general coupling formula in equation (13) of Section 5.1., and substitute M_1, ψ_1, $-M_2$ and $-\psi_2$ for M_a, ψ_a, M_b and ψ_b. We also set the multiplying factors K_v, K_h in equation (13) equal to ρ_v, ρ_h respectively and derive the square of the modulus as follows:

$$F_g = \frac{|\rho_v(\cos\psi_1 + jM_1 \sin\psi_1)(\cos\psi_2 - jM_2 \sin\psi_2) - \rho_h(\sin\psi_1 - jM_1 \cos\psi_1)(\sin\psi_2 + jM_2 \cos\psi_2)|^2}{(1 + M_1^2)(1 + M_2^2)} \qquad (6)$$

It should be emphasized that in this expression (as also in the expression for F_s in the previous section) the parameters M_1, ψ_1, M_2 and ψ_2 always apply to the polarization of the ordinary mode calculated for the downcoming wave approaching the Earth reflexion point from the terminals of the path.

If the substitution $\rho_v = 1$, $\rho_h = -1$ applicable to a sea reflexion is made in equation (6) it becomes equivalent to the result for F_s given by equation (3), as would be expected.

In the case of ground of sufficient uniformity in depth† the reflexion coefficients may be deduced from the ground constants. Fig. 6 shows the amplitude and phase of the reflexion coefficient for vertically polarized waves and grounds of

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*The positive direction for the horizontal component is assumed to be towards the left. This arises from the convention explained in Section 5.1., whereby the vertical axis is taken as the x axis and the horizontal axis as the y axis.

†The ground should be uniform for not less than the 'penetration depth' for a normally-incident wave. This is the depth at which the amplitude of the wave falls to $1/e$ (37%) of its original value. It depends on the ground constants, typical values for 1 Mc/s being 5 m and 18 m for 'good' and 'poor' soil conductivity. For sea water the penetration depth is 0·25 m.

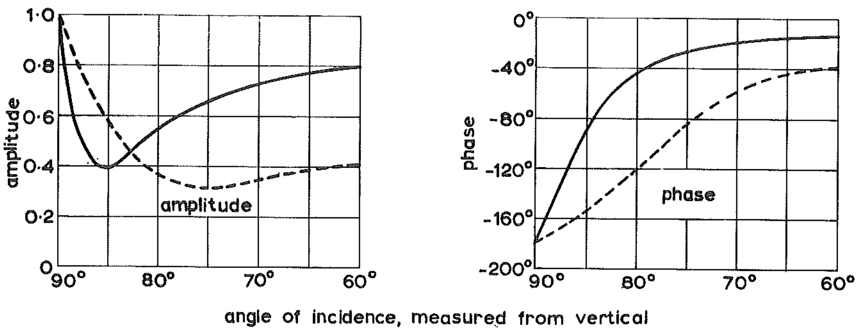

Fig. 6 - Ground reflexion coefficient for vertically polarized waves

——— Good conductivity (ϵ_{rel} = 10, σ = 10^{-2} mhos/m)

- - - Poor conductivity (ϵ_{rel} = 4, σ = 10^{-3} mhos/m)

Frequency 1 Mc/s

good and poor conductivities. It will be seen that the amplitude of the reflexion coefficient has a minimum at the so-called Brewster angle, which at medium frequencies lies within the range of angles which are of interest for long-distance propagation. The amplitude of the reflexion coefficient for horizontally polarized waves is approximately unity over this range of angles for all ground conductivities. At low angles near the Brewster angle, therefore, the horizontally polarized component represents the principal contribution to the energy reflected from the ground. The phase of the reflexion coefficient for horizontally polarized waves is approximately 180°.

3. DISCUSSION

The main purpose of this report is to indicate the method of calculating the effect of polarization on the transmission of medium frequencies over long distances at night, on the assumption that only one mode is important and that the form of its polarization is known.

As an indication of when a serious reduction of the m.f. sky-wave at long ranges might be expected, the following special cases are worth mentioning:

(i) When the dip angle is small, E-W paths of all lengths (i.e. whether single- or multi-hop) will suffer a serious loss as a result of poor coupling with the vertically polarized transmitting and receiving aerials. This is because the ordinary-wave polarization is approximately linear and horizontal.

(ii) When the dip angle is small and when *multi-hop paths over sea* are involved, serious loss occurs for N-S paths. In this case the polarization of the downcoming wave is approximately circular but, after reflexion from the sea, has the wrong sense of rotation for efficient coupling with the ordinary mode on re-entering the ionosphere.

(iii) When the dip angle is near 45° and when *multi-hop paths over sea* are again involved, serious loss occurs for E-W paths. Here the polarization is approximately linear but tilted at 45° and, when reflected in the sea, is at right angles to the linearly polarized wave corresponding to the ordinary mode on re-entering the ionosphere.

The precise value of coupling loss for these and other cases is a function of many parameters as already discussed. If the conditions are such that only one mode of propagation in the ionosphere is significant, the procedure described in Section 2 is valid.

We may summarize this procedure for calculating the total coupling loss as follows. In the case of a single-hop path we first determine the polarizations associated with the two terminals. The loss, expressed as a power ratio, is then determined as the product of two 'terminal loss' factors F_t and F_r as described in Section 2.3. In the case of multi-hop paths, we must also determine the polarizations associated with each Earth reflexion point and derive the additional loss factor(s) F_s or F_g for sea or ground as appropriate, using the formulae given in Section 2.4. For a two-hop path over sea, for example, the total power loss factor is $F_t F_s F_r$; for a three-hop path entirely over ground we would have $F_t F_g F'_g F_r$ where F_g and F'_g apply to the two ground reflexion points.

The results of the calculation may be applied in two ways. First, if the other main cause of loss is known, viz. the attenuation in the ionosphere, an absolute estimate of the field strength relative to free space conditions may be made. Second, if the attenuation data be not well established, or if other propagation phenomena such as diffuse reflexion may give rise to errors, it is possible to refer to standard propagation curves of field strength versus distance for a particular zone. Such curves are generally based on the average of many paths and an estimate may be made of the average value of coupling loss which would apply to these paths. For example, in Europe the coupling loss is about 2 dB and 10 dB for one-hop and two-hop paths respectively. Therefore the correction to be applied to curves such as those published by the EBU[3] when applying them to a given path will be the amount by which the estimated coupling loss for that path differs from the average value, namely from 2 dB for a one-hop path or 10 dB for a two-hop path in the example given.

The form of polarization given by equations (13) or (14) in the Appendix, Section 5.2., is an approximation based on ray-theory, and the curves of the report illustrate this approximate polarization when the frequency is close to the gyromagnetic frequency in the E-region. Further work is required to determine

(a) the conditions under which the consideration of a single mode is adequate,

(b) whether it is practicable to derive more accurate curves of the polarization of the mode or modes of importance based on a wave treatment, and

(c) whether curves can be conveniently presented to cover the wide range of different parameters needed for prediction of field strengths in various medium-frequency applications.

4. REFERENCES

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2. Eckersley, T.L. and Millington, G.: 'The Limiting Polarization of Medium Waves reflected from the Ionosphere', The Proceedings of the Physical Society, January 1939, **51** , 283, p. 110.

3. Ebert, W.: 'Ionospheric Propagation on Long and Medium Waves', EBU Technical Monograph, No. 3081, June 1962.

5. APPENDIX

5.1. The Degree of Coupling between Elliptically Polarized Waves

A general case is considered in which a given elliptical polarization may be resolved into two other orthogonal elliptical polarizations, with the object of determining the power coupling factor into one of these other polarizations. Allowance will also be made for a possible modification of the amplitude of the components parallel to two axes at right angles in order to cover the application to ground reflexion discussed in Section 2.4.2.

When an elliptically polarized wave entering the ionosphere is being considered, it must be resolved into two modes which match the characteristic modes of the medium. Provided that the electron collision frequency is low, these modes are orthogonal, that is to say if one mode is represented by an ellipse the other mode is represented by an ellipse of the same axial ratio, but with the opposite sense of rotation and with the major axis at right angles to that of the first ellipse. These modes have the property that, within a portion of the medium over which the physical conditions remain constant, one mode does not couple with the other, and each mode propagates without changing the form of polarization. There may, of course, be a gradual change of the polarization of a mode along a path of many wavelengths due to a slow change in propagation direction or other parameters, but this effect does not involve coupling losses of the type to be considered here.

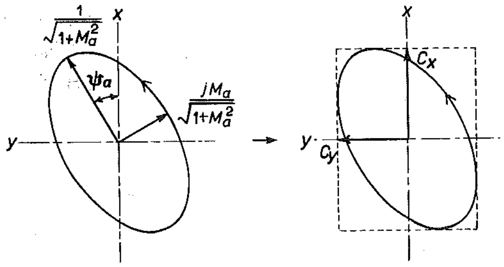

Fig. 7 - Resolution of elliptical polarization into orthogonal linear components

First consider an incident elliptically polarized wave with components of $(1 + M_a^2)^{-\frac{1}{2}}$ on the major axis and $jM_a(1 + M_a^2)^{-\frac{1}{2}}$ on the minor axis, as shown in Fig. 7. These amplitudes have been chosen to have a ratio of M_a and a total power equivalent to that of a plane-polarized wave of unit amplitude. If the major axis is at angle ψ_a from the x-axis,* measured anti-clockwise, the wave may be resolved into x and y components whose values are given by

$$C_x = \frac{\cos\psi_a + jM_a\sin\psi_a}{(1 + M_a^2)^{\frac{1}{2}}} \tag{7}$$

$$C_y = \frac{\sin\psi_a - jM_a\cos\psi_a}{(1 + M_a^2)^{\frac{1}{2}}} \tag{8}$$

*The x-axis has been placed in a vertical direction as this has been taken as the reference axis for ψ. This is for consistency of notation; in certain problems, e.g. with loop aerials, it is convenient to discuss the polarization of the magnetic field component, in which case the polarization ellipse is similar but rotated 90°, and the reference axis becomes horizontal.

It is next convenient to consider the converse problem of resolving an incident wave which is linearly polarized—along the x-axis, say—into an elliptically polarized component of prescribed form and its orthogonal form. Let the ellipse of the prescribed polarization have an axial ratio and tilt of M_b and ψ_b. Fig. 8(a) illustrates the simple case when $\psi_b = 0$. It may be seen that the axial components shown in the figures for the prescribed ellipse, P, and the orthogonal ellipse, Q, sum to give the linear component of unit amplitude. It will also be noted that the sum of the powers in the elliptically polarized waves is equal to that of the incident wave.

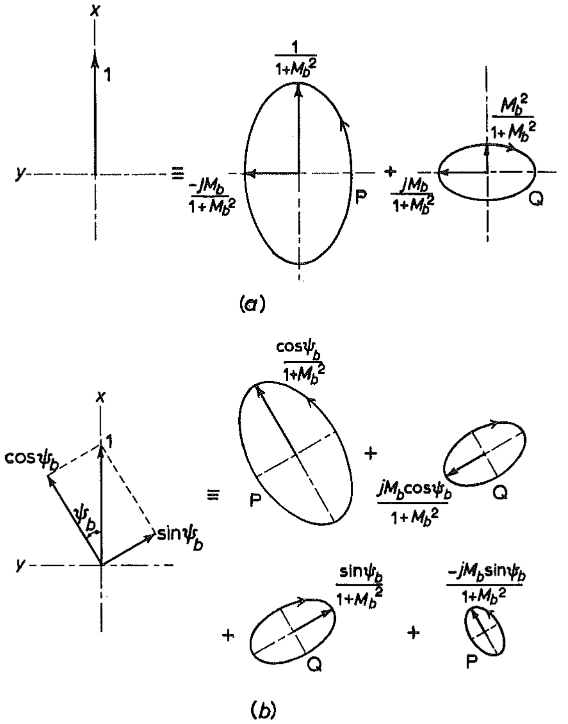

Fig. 8 - Resolution of linear polarization into orthogonal elliptical components

(a) Axis of ellipse parallel to linear polarization

(b) Axis of ellipse at arbitrary angle

The result for the more general case illustrated in Fig. 8(b) may now be derived. The vertically polarized wave is first resolved into linear components, parallel and perpendicular to the axes of the prescribed polarization; these are then resolved into the prescribed polarization P and the orthogonal form Q. For simplicity only the amplitudes of the major axes are shown in Fig. 8(b) for each of the four elliptical components. The first two of these are regarded as generated from the linear component $\cos\psi_b$ of the incident plane polarized component, while the second two are derived from the linear component $\sin\psi_b$. In the latter case it will be observed that the senses of rotation of the ellipses have been reversed relative to the case shown in Fig. 8(a). This allows direct addition in pairs to give two resultant elliptical components whose major axis components are

$$C_p = \frac{\cos\psi_b - jM_b\sin\psi_b}{1 + M_b^2} \tag{9}$$

$$C_q = \frac{\sin\psi_b - jM_b\cos\psi_b}{1 + M_b^2} \tag{10}$$

The component C_p represents the resolution of the incident linearly polarized wave of Fig. 8(b) into a prescribed elliptical form with the major axis oriented ψ_b from the plane of polarization of the incident wave. C_q gives the orthogonal component.

The results obtained when the resolved parts of one form of elliptical polarization generate another form can now be seen. The incident wave is first resolved into x and y components as shown in Fig. 7; we may assume for generality that they are subject to multiplying factors K_v and K_h respectively. The linearly polarized components therefore have amplitudes K_vC_x and K_hC_y, where C_x and C_y are given by equations (7) and (8). The vertical component gives rise to a prescribed polarization whose major axis component is equal to $K_v C_x C_p$; this process is illustrated in Fig. 8(b). The horizontal component also generates the prescribed polarization, but in this case it may be shown that the major axis component is given by

$$K_h C_y \cdot \frac{\sin\psi_b + jM_b\cos\psi_b}{1 + M_b^2}$$

the final term being the expression C_p with angle $(\psi_b - 90°)$ substituted for ψ_b. The total major axis component C_r of the prescribed polarization is therefore equal to

$$C_r = K_v C_x C_p + K_h C_y (\sin\psi_b + jM_b\cos\psi_b)/(1 + M_b^2) \tag{11}$$

The minor axis component is equal to $jM_b C_r$.

The power in an elliptically polarized wave is proportional to the sum of the squares of the amplitudes of its major and minor axis components. For the incident wave of Fig. 7 this quantity is equal to unity and for the generated wave it is equal to $(1 + M_b^2)|C_r|^2$. The power coupling factor is therefore equal to

$$F = (1 + M_b^2)|C_r|^2 \tag{12}$$

Substitution of the component values given in equations (7), (8), (9) and (11) leads to the following result:

$$F = \frac{|K_v(\cos\psi_a + jM_a\sin\psi_a)(\cos\psi_b - jM_b\sin\psi_b) + K_h(\sin\psi_a - jM_a\cos\psi_a)(\sin\psi_b + jM_b\cos\psi_b)|^2}{(1 + M_a^2)(1 + M_b^2)} \tag{13}$$

If the special case in which $K_v = K_h = 1$ is considered the numerator simplifies to

$$(1 + M_a M_b)^2 \cos^2(\psi_a - \psi_b) + (M_a + M_b)^2 \sin^2(\psi_a - \psi_b)$$

which is the result quoted in equation (1). The application discussed in Section 2.4.2. requires the more general form of equation (13).

5.2. Formulae for Calculating the Polarization of the Ordinary Wave

The propagation direction is required in order to determine the ellipticity of polarization. The major axis of the polarization ellipse for the electric field lies in a plane containing the direction of the total component of the Earth's magnetic field and the ray direction.* The appropriate definitions and formulae are as follows:

Given : D = Magnetic dip angle at the reflexion point (angle of Earth's magnetic field relative to the horizontal). (-90° to +90°, positive to the north of the magnetic equator.)

ϕ_m = Magnetic bearing of transmitter relative to observing receiver, measured east of north. (0° to 360° or -180° to +180°.)

θ = Ionospheric angle of incidence (angle of downcoming ray-direction relative to the vertical at the reflexion point). (0° to 90°.)

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*This is strictly true only when the collisional frequency of the electrons is small; the formulae stated in this Appendix apply to this case. At medium frequencies the effect of electron collisions on the polarization characteristics is believed to be unimportant.

To find: θ_m = Angle between the downcoming ray-direction and the Earth's magnetic field. (0° to 180°.)

ψ = Orientation of the ellipse of the electric field polarization expressed as the rotation about the ray-direction from a position in which the minor axis is horizontal. Looking downwards along the ray-direction, this rotation is anticlockwise if ψ is positive and clockwise if ψ is negative. (-90° to +90°.)

Formulae:

$$\cos\theta_m = \cos\theta \sin D - \sin\theta \cos D \cos\phi \tag{14}$$

$$\cot\psi = \tan D \operatorname{cosec}\phi_m \sin\theta + \cot\phi_m \cos\theta \tag{15}$$

The axial ratio of the polarization ellipse depends on frequency and the angle θ_m defined above.

Given: f = operating frequency

f_y = gyro-magnetic frequency at reflexion point

To find: M = axial ratio of ellipse (-1 to +1)

Formula: $M = \tan \omega/2$ (16)

where $\cot\omega = f_y \sin\theta_m \tan\theta_m/2f$ (ω in the range -90° to +90°)

Note that if $90° < \theta_m \leqslant 180°$, the values of ω and M are negative. A positive value means that the sense of polarization is anticlockwise looking down the ray, while a negative value means that the sense is clockwise.

If the operating frequency is equal to the gyro frequency the expression for the axial ratio simplifies to

$$M = \cos\theta_m \tag{17}$$

BRH

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